

Objective: To manipulate arrays iteratively.

- Given vectors  $A : int^n$  and  $B : int^n$ , add them up to give vector  $C : int^n$ . Use **map**.
- Given array  $A : int^n$ , find sum of its elements. Use **fold**.  
We can also compute  $\Sigma(A[i]^2)$  and use this to find standard deviation.
- **Mapfold** combines the map and the fold.

# Map

- **Example:** Adding two 3-dimensional vectors  $a, b: \text{real}^3$  to get  $c: \text{real}^3$ .

Method: Use  $\pm$  pointwise on every index putting the result in the output array.

- $c = \text{map}\langle\langle 3 \rangle\rangle(+)$  ( $[1, 3, 5]$ ,  $[4, 3, -2]$ ) gives  $[5, 6, 3]$

- In general  $\text{map}\langle\langle n \rangle\rangle(F)$  ( $x_1, \dots, x_m$ ) returns  $(y_1, \dots, y_k)$

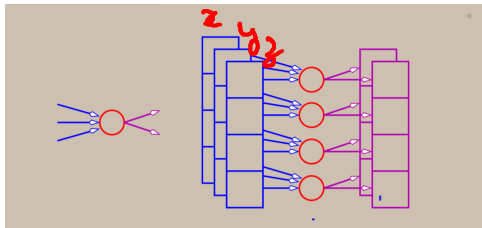
Here  $F : (t_1 \times \dots \times t_m) \rightarrow (t'_1 \times \dots \times t'_k)$ .

Also,  $x_i : t_i^n$  for  $1 \leq i \leq m$  and  $y_j : t'_j^n$  for  $1 \leq j \leq k$ .

- Expression  $\text{map}\langle\langle n \rangle\rangle(F)$  has type

$(t_1^n \times \dots \times t_m^n) \rightarrow (t'_1^n \times \dots \times t'_k^n)$ .

$\text{map}\langle\langle 3 \rangle\rangle(F)$   
 $(x, y, z)$   
 $\rightarrow (u, v)$

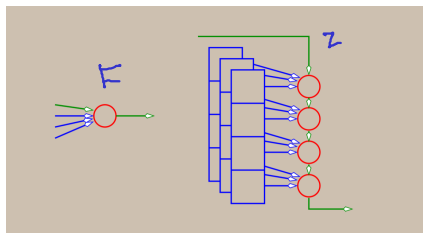


Here  
 $m = 3$   $k = 2$   
 $n = 4$

# Fold

- **Example:** Finding sum of array of 4 elements  $a:\text{int}^4$  to get  $c:\text{int}$ .  
Method: Use  $+$  pointwise on every index accumulating the sum.
- $c = \text{fold}\langle\langle 4 \rangle\rangle(+)([1,3,5,7],0)$  gives **16**
- In general  $\text{fold}\langle\langle n \rangle\rangle(F)(x_1, \dots, x_m, z)$  returns  $y$   
Here  $F : (t_1 \times \dots \times t_m \times t) \rightarrow t$ .  
Also,  $x_i : t_i^n$  for  $1 \leq i \leq m$  and  $z, y : t$ .
- Expression  $\text{fold}\langle\langle n \rangle\rangle(F)$  has type  $(t_1^n \times \dots \times t_m^n \times t) \rightarrow (t)$ .

$m=3$



Handwritten calculation showing the accumulation of the sum:

$$\begin{aligned} 0 &\rightarrow + \rightarrow 1 \\ 3 &\rightarrow + \rightarrow 4 \\ 5 &\rightarrow + \rightarrow 9 \\ 7 &\rightarrow + \rightarrow 16 \end{aligned}$$

# Example

$$a[0] * b[0] + a[1] * b[1] + \dots + a[n-1] * b[n-1]$$

$\downarrow$                        $\downarrow$                                        $\downarrow$

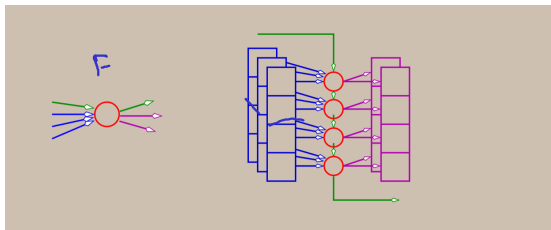
$z[0]$                        $z[1]$                                        $b[n-1]$

Find dot product of two  $n$ -dimensional vectors.

```
node dotproduct<<n:int>>(a:real^n; b:real^n) returns (c:real)
var z:real^n
let
  z = map<<n>>(*) (a,b);
  c = fold<<n>>(+)(z,0);
tel
```

# Mapfold

- Example: Adding two 3-dimensional vectors  $a, b: \text{real}^3$  AND getting their dot-product  $c: \text{real}^3; \text{dot}: \text{real}$
- node  $F(x, y, \text{accin}: \text{real})$  returns  $(z, \text{accout}: \text{real})$   
let  $z = x + y; \text{accout} = \text{accin} + (x * y); \text{tel}$
- $c = \text{mapfold}\langle\langle 3 \rangle\rangle(F) ([1, 3, 5], [4, 3, -2], 0)$  gives  $[5, 6, 3], 13$
- In general  $\text{mapfold}\langle\langle n \rangle\rangle(F) (x_1, \dots, x_m, \text{init})$  returns  $(y_1, \dots, y_k, \text{acc})$   
Here  $F : (t_1 \times \dots \times t_m \times t) \rightarrow t'_1 \times \dots \times t'_k \times t$ .  
Also,  $x_i : t_i^n$  for  $1 \leq i \leq m$  and  $y_j : t'_j^n$  for  $1 \leq j \leq k$  with  $\text{init}, \text{acc} : t$ .



mdtx

0  $F(1, 4, 0)$   
= 5, 4  
1  $F(3, 3, 4)$   
= 6, 13  
2  $F(5, -2, 13)$   
= 3, 3

- Features for writing large and complex programs.
- Records, Arrays, Array slices, Global types and constants, Parameterized nodes.
- Array iterators: map, fold and mapfold.