

Objective: To manipulate arrays iteratively.

- Given vectors $A : int^n$ and $B : int^n$, add them up to give vector $C : int^n$. Use **map**.
- Given array $A : int^n$, find sum of its elements. Use **fold**.
We can also compute $\Sigma(A[i]^2)$ and use this to find standard deviation.
- **Mapfold** combines the map and the fold.

Map

- **Example:** Adding two 3-dimensional vectors $a, b: \text{real}^3$ to get $c: \text{real}^3$.

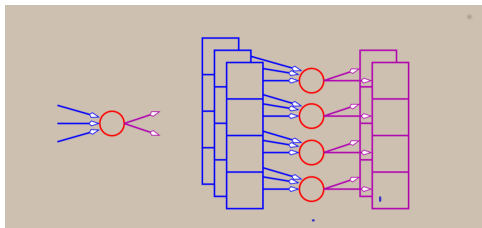
Method: Use \pm pointwise on every index putting the result in the output array.

- $c = \text{map}\langle\langle 3 \rangle\rangle(+)([1, 3, 5], [4, 3, -2])$ gives $[5, 6, 3]$
- In general $\text{map}\langle\langle n \rangle\rangle(F)(x_1, \dots, x_m)$ returns (y_1, \dots, y_k)

Here $F: (t_1 \times \dots \times t_m) \rightarrow (t'_1 \times \dots \times t'_k)$.

Also, $x_i: t_i^n$ for $1 \leq i \leq m$ and $y_j: t'_j^n$ for $1 \leq j \leq k$.

- Expression $\text{map}\langle\langle n \rangle\rangle(F)$ has type $(t_1^n \times \dots \times t_m^n) \rightarrow (t'_1^n \times \dots \times t'_k^n)$.

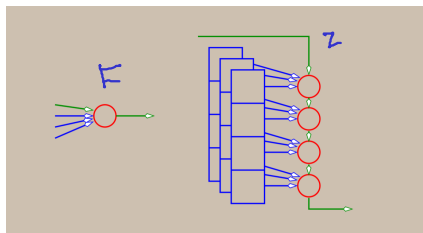


Here
 $m=3$ $k=2$
 $n=4$

Fold

- **Example:** Finding sum of array of 4 elements $a:\text{int}^4$ to get $c:\text{int}$.
Method: Use $+$ pointwise on every index accumulating the sum.
- $c = \text{fold}\langle\langle 4 \rangle\rangle(+)$ ($[1,3,5,7],0$) gives **16**
- In general $\text{fold}\langle\langle n \rangle\rangle(F)$ (x_1, \dots, x_m, z) returns y
Here $F : (t_1 \times \dots \times t_m \times t) \rightarrow t$.
Also, $x_i : t_i^n$ for $1 \leq i \leq m$ and $z, y : t$.
- Expression $\text{fold}\langle\langle n \rangle\rangle(F)$ has type $(t_1^n \times \dots \times t_m^n \times t) \rightarrow (t)$.

$m=3$



0
1 → + → 1
3 → + → 4
5 → + → 9
7 → + → 16

Example

$$a[0] * b[0] + a[1] * b[1] + \dots + a[n-1] * b[n-1]$$

\downarrow \downarrow \downarrow

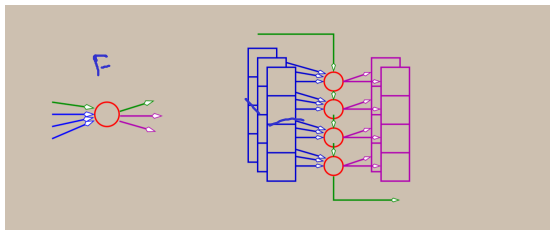
$z[0]$ $z[1]$ $b[n-1]$

Find dot product of two n -dimensional vectors.

```
node dotproduct<<n:int>>(a:real^n; b:real^n) returns (c:real)
var z:real^n
let
  z = map<<n>>(*) (a,b);
  c = fold<<n>>(+)(z,0);
tel
```

Mapfold

- Example: Adding two 3-dimensional vectors $a, b: \text{real}^3$ AND getting their dot-product $c: \text{real}^3; \text{dot}: \text{real}$
- node $F(x, y, \text{accin}: \text{real})$ returns $(z, \text{accout}: \text{real})$
let $z = x + y; \text{accout} = \text{accin} + (x * y); \text{tel}$
- $c = \text{mapfold}\langle\langle 3 \rangle\rangle(F) ([1, 3, 5], [4, 3, -2], 0)$ gives $[5, 6, 3], 13$
- In general $\text{mapfold}\langle\langle n \rangle\rangle(F) (x_1, \dots, x_m, \text{init})$ returns $(y_1, \dots, y_k, \text{acc})$
Here $F : (t_1 \times \dots \times t_m \times t) \rightarrow t'_1 \times \dots \times t'_k \times t$.
Also, $x_i : t_i^n$ for $1 \leq i \leq m$ and $y_j : t'_j^n$ for $1 \leq j \leq k$ with $\text{init}, \text{acc} : t$.



mdtx

0 $F(1, 4, 0)$
= 5, 4
1 $F(3, 3, 4)$
= 6, 13
2 $F(5, -2, 13)$
= 3, 3

- Features for writing large and complex programs.
- Records, Arrays, Array slices, Global types and constants, Parameterized nodes.
- Array iterators: map, fold and mapfold.